Growth Points That Describe Young Children’s Learning in the Counting, Place Value, Addition & Subtraction and Multiplication & Division Domains

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Introduction

Children begin to learn number concepts and skills from a very early age as they interact with their families and explore their environments. Before beginning school, most children know the names of many numbers, can recite sequences of number names and attach a numerical value to a group. Indeed, many children have acquired a degree of number sense which allows them to solve intuitively the problems they encounter as part of daily living and learning. However, this is not the case for all children. One important challenge for teachers is to provoke the number learning of all children.

In order to meet this challenge, we need to plan learning opportunities for children based on a clear understanding of: (1) children’s mathematical thinking; (2) the possible course of children’s mathematical learning and the major growth points that describe children’s number learning; (3) instructional sequences that provoke and support mathematical learning; and (4) the difficulties that some children experience when working with numerical situations. This paper explores the key growth points that describe children’s mathematical learning in the domains of Counting, Place Value, Addition and Subtraction Strategies, and Multiplication and Division Strategies. The role of visualisation in children’s number learning will also be explored.

Using Growth Points to Describe Children’s Learning

The growth points described in this paper were developed as part of the Early Numeracy Research Project (ENRP, Clarke, Gervasoni, & Sullivan, 2000), and provided a basis for the construction of tasks for one to one interviews that enable teachers to identify the growth points children have reached in their mathematical learning. Use of open-ended tasks within mathematics lessons, as well as specifically designed on-going assessment tasks provide further insights for teachers about children’s understanding of number.

We have come to describe growth points as key "stepping stones" along paths to mathematical understanding. However, we do not claim that all growth points are passed by every student along the way. Also, the growth points should not be regarded as necessarily discrete. As with Wright’s (1998) learning framework, the extent of the overlap is likely to vary widely across young children, and "it is insufficient to think that all children’s early arithmetical knowledge develops along a common developmental path" (p. 702).

Growth Points are useful because they help teachers:
• understand how children learn;
• assess and monitor children's growth in understanding;
• identifying children who are at risk;
• identify the zone of proximal development for children’s learning; and
• plan and target teaching so that we can identify the experiences that will most effectively help children to reach the next growth point in their mathematical learning.

The ENRP growth points therefore provide teachers with a useful framework for reflecting on and evaluating the strategies a child uses to solve numerical problems. This information can be used as a starting point in planning future instructional activities that meet the learning
needs of each child and provoke and support children move towards the next growth point in their learning.

COUNTING

The development of children’s number sense is of paramount importance in the first years of schooling. Number sense is about being comfortable and ‘at home’ with numbers. It is also about being effectively and efficiently able to work with numbers to solve the problems encountered as part of daily living. Initially, children use counting strategies to solve the numerical problems they encounter. Counting forwards in ones, or even in twos, fives and other multiples, are strategies that may be used to solve addition problems. Similarly, backwards counting may be used to solve subtraction problems.

Counting becomes a powerful tool for children to solve problems when they can both accurately count a collection, and can count-on and count back from any number by ones, twos, fives and other multiples. Prior to reaching these growth points, children learn to count by rote. In English, this is a difficult task as there is no easily generalisable pattern to follow.

In contrast to English, a feature of Asian languages is the fact that the place value of number names is transparent. For example, an English translation of counting in Vietnamese beyond ten is: ten one, ten two, ten three, ten four, ten five, ten six, ten seven, ten eight, ten nine, two tens, two ten one and so forth. Counting in Asian languages follows a generalisable pattern that is easy to learn, and the value of the number is easy to interpret from the number name. Counting from zero to one hundred in English requires combinations of twenty nine words, as opposed to only eleven words in Vietnamese and other Asian languages. European languages such as English and Italian, use new words beyond ten, such as eleven, twelve, thirteen, fourteen, twenty and thirty. These number names make it very difficult for children to establish generalisable counting patterns and interpret the value of these number words.

Teachers often try to assist children to interpret ‘teen’ and ‘ty’ as ten to ease this problem for English speaking children, but the fact remains that learning to count and interpret number words in English is more difficult than performing the same task in an Asian language.

Counting Growth Points

In order to plan learning opportunities for children that will assist them to learn to count, it is useful to consider the possible course or trajectory that describes learning to count. The Curriculum and Standards Framework: Mathematics (Board of Studies, 1999) describes the curriculum focus statements and broad learning outcomes for children’s mathematical learning. However, at times teachers need to focus in more detail on the major growth points associated with the learning trajectory. Assessment tasks that suggest the positions children have reached along this learning trajectory and identify the growth points they have developed, enable teachers to provide instructional activities that will provoke and support children move towards the next growth point in their learning.

The ENRP Counting Growth Points describe the development of children’s counting by ones, as well as by other multiples from two to ten. As with the stages described by Wright, Martland and Stafford (2000), these growth points are concerned with children’s production of number name sequences. However, in contrast to Wright et al., the ENRP growth points are also concerned with children making the count-to-cardinal transition in word meaning described by Fuson (1992) so that children are able to think about the number sequence to solve problems. The growth points do not describe children’s use of counting in addition, subtraction, multiplication and division problem solving situations. Such strategies are described in growth points pertaining to the Addition and Subtraction, and Multiplication and Division domains of the ENRP Learning and Assessment Framework.
Figure 1 lists the ENRP Growth Points that describe the learning trajectory as children learn to count.

0. Not apparent.
   Not yet able to state the sequence of number names to 20.
1. Rote counting
   Rote counts the number sequence to at least 20, but is not yet able to reliably count a collection of that size.
2. Counting collections
   Confidently counts a collection of around 20 objects.
3. Counting by 1s (forward/backward, including variable starting points; before/after)
   Counts forwards and backwards from various starting points between 1 and 100; knows numbers before and after a given number.
4. Counting from 0 by 2s, 5s, and 10s
   Can count from 0 by 2s, 5s, and 10s to a given target.
5. Counting from x (where x > 0) by 2s, 5s, and 10s
   Given a non-zero starting point, can count by 2s, 5s, and 10s to a given target.
6. Extending and applying counting skills
   Can count from a non-zero starting point by any single digit number, and can apply counting skills in practical tasks.

A possible extension to the ENRP growth points is
   * Counting from X by other quantities, e.g., fractions, decimals and larger whole numbers.

Figure 1. ENRP Counting growth points

PLACE VALUE

It is fascinating to observe children as they begin to read, write and interpret numerals. Here again, the English language makes this a challenging task. Many young children initially write fourteen as 41 or sixteen as 61. Perhaps these children are over-generalising spelling strategies? In attempting to write fourteen they say the word and hear the sound four at the beginning. These children know the symbol for four and so begin with 4 when writing the numeral for fourteen. These children, however, do not yet recognise that place value conventions are used to write and read numerals, not spelling conventions. Similar observations are made when young children write larger numbers such as one hundred and twenty-four. It is common to see children write this numeral as 100204. These children know that the symbol for one hundred is 100, the symbol for twenty is 20 and the symbol for four is 4. So when they hear one hundred and twenty-four, they write the known symbols for the sounds they hear, just as they do when spelling words. Again, this is a sign that these children have not yet learnt the place value conventions of our number system.

A similar situation occurs when children begin to read two and three digit numbers. A number such as 41 may be read as fourteen, 57 may be read as five and seven, or 326 as three and twenty-six. Such observations indicate that a child does not yet understand the place value conventions for reading numbers. Exploring the conventions of reading, writing and interpreting numbers is an important activity in junior primary school classrooms.

Developing number concepts

Number is a concept that we construct from our experiences. In coming to interpret a number, we have to bring together three pieces of information: a known collection; the word that represents the numerical value of the collection; and the symbol used to record this number word (see Figure 2). These three ideas can be conceptualised as a triad comprising six relationships. For example, for the number five children learn to

![Figure 2: Number triad comprising six relationships.](image)
relate: a collection of five objects to the number name for five; the number name for five to the collection; the number word for five to the symbol for five; the symbol to the number word; the symbol for five to a collection of five; and a collection of five to the symbol for five. These six relationships are developed for each new number a child learns. Many classroom experiences are geared towards helping children understand these relationships. Some examples follow:

1. Here are some counters (pile of 5-10 or 20-30). How many do you think there might be? Check to find out. Type this number on a calculator.
2. Get sixteen counters from a container. Write this number.
3. Name this number (show the numeral 6). Clip this many pegs to the card.
4. How many stickers are on this sheet? How did you work that out?
   Can you write that number?
   If I added another row of stickers, how many would there be? Type this number on a calculator.
   Cover two rows of stickers.
   How many stickers have I covered?

**Learning the conventions of our number system**

Initially when children work with collections of objects greater than ten, they establish the numerical value of the collection through counting. The collection is perceived as a collection of single objects. Thus a pile of thirty-six blocks is perceived as 36 individual blocks, not as a collection of 3 tens and six ones. However, to write the numeral for a collection of thirty-six, we follow place value conventions which focus on the number of tens and ones in the collection. This culturally based convention must be explicitly pointed out to most children. In learning to read, write and interpret numerals, children learn the following characteristics of our number system: combinations of only ten digits (0-9) are used to represent all numbers; the value of a digit depends on its place in the numeral; zero performs the function of holding a place when it has no value; and the value of each digit in a numeral is added to determine the numeral’s total value.

**Place Value Growth Points**

In order to plan learning opportunities for children that will promote the development of place value ideas, we need to consider the possible course or trajectory of children’s place value learning and the major growth points that describe this learning. Assessment tasks that suggest the positions children have reached along this learning trajectory and identify the growth points they have developed, enable teachers to provide instructional activities that will provoke and support children move towards the next growth point in their learning. Figure 3 lists growth points that describe the learning trajectory as children learn to read, write, order and interpret numerals.

0. Not apparent
   Not yet able to read, write, interpret and order single digit numbers.
1. Reading, writing, interpreting, and ordering single digit numbers
   Can read, write, interpret and order single digit numbers.
2. Reading, writing, interpreting, and ordering two-digit numbers
   Can read, write, interpret and order two-digit numbers.
3. Reading, writing, interpreting, and ordering three-digit numbers
   Can read, write, interpret and order three-digit numbers.
4. Reading, writing, interpreting, and ordering numbers beyond 1000
   Can read, write, interpret and order numbers beyond 1000.
5. Extending and applying place value knowledge
   Can extend and apply knowledge of place value in solving problems.

* A possible extension to the ENRP growth points is
* reading, writing and interpreting number involving millions, decimals and fractions.

*Figure 3. ENRP Place value growth points*
Developing Place Value Ideas

One way to help children learn about place value conventions is to play a game such as Race Past 100. This game requires one die or spinner and a large tub of materials such as lima beans, counters, blocks or icy pole sticks. The child rolls the die, takes the number of objects from the tub represented by the dots on the die and forms a pile of objects. This process is repeated until a large pile is produced or a certain period of time has elapsed, such as three minutes. The child is then asked to determine as quickly as possible the number of objects that are in the pile. Children generally determine this through counting by ones. However once this activity is repeated several times, children begin to develop more powerful strategies. They may group the objects in their pile and use skip counting to determine the number in the collection. Usually some children begin to group their pile into tens. Highlighting this strategy during class mathematical discussions following this game gives the teacher an opportunity to point out that determining the number of hundreds, tens and ones in a collection helps us to write the numeral to represent the collection.

Similarly, when we read numerals, the different digits tell us about the number of hundreds, tens and ones this numeral represents. Thus children can be encouraged to play Race Past 100 from now on using grouping by ten strategies. Counters or beans can be placed on ten frames, blocks can be joined to make sticks of ten, and icy pole sticks can be bundled into tens using rubber bands. When one hundred is reached, the 10 tens may be placed into a hundreds sack.

Sometimes children invent other ways to represent ten or one hundred. For example, they may use different coloured counters to represent the tens or hundreds. Such non-proportional modeling is also used in an abacus and is a more abstract way of representing tens and hundreds. The reasoning behind using this strategy highlights an important mathematical idea which can be a focus of a class discussion. It may also be useful for the teacher to provide place value mats to help children organise their materials when playing Race Past 100. Having determined the value of the pile produced, the children can be challenged to place numeral cards on the Place Value Chart (see Figure 4) to indicate how many hundreds, tens and ones they have collected, thus creating the numeral to represent the collection.

<table>
<thead>
<tr>
<th>Hundreds</th>
<th>Tens</th>
<th>Ones</th>
</tr>
</thead>
<tbody>
<tr>
<td>[Diagram of abacus with 1, 3, and 7]</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Figure 4: Representation of 137 beans on a place value board.

The Place Value Chart, numeral cards and materials can also be used by children to practise making collections to represent numerals. One child could place two or three numerals on the place value chart and then challenge a partner to place on the chart a collection of objects to represent the numeral. This game can be extended to include tenths and hundredths to assist children read, write and interpret decimal numbers.

Numerals cards may be used in many ways to help children learn place value ideas. For example, children may be challenged to draw three numeral cards from a pack and then arrange the cards to form numerals to represent the largest or smallest possible numbers. The number of cards used to form numerals can be increased as children learn to interpret decimal numbers and numbers involving thousands.
Addition & Subtraction Strategies

Traditionally, children have been taught to identify the numerical value of a group through counting by ones. However, an emphasis on such counting means that many children remain dependent on less powerful and inefficient counting strategies to solve basic problems.

Provoking children to move beyond using counting strategies is an important aspect of primary school mathematics programs. More powerful strategies for solving mathematical problems include: counting on, counting back, counting up from, counting down to, skip counting, using known facts, using doubles facts, using near doubles, building to five, compensation, subitising (recognising visual patterns), using addition to solve subtraction and commuting an addition.

In order to provoke children develop more powerful number strategies for solving numerical problems, teachers first need to explore the mathematical thinking and strategies each child uses to solve problems. The strategies used by children indicate their current understanding of number and their power in working with number. Many children fail to develop an increasing power in working with number through dependence on less powerful and inefficient methods, such as counting by ones. However, children will not always use the most powerful strategy in their repertoire for solving a problem. Indeed, they often use less powerful strategies to solve simple problems. It is therefore important to give children the opportunity to solve problems at a level of difficulty that challenges their current understanding and provokes them to use more powerful strategies. Children should also be encouraged to explain their reasoning for using a particular strategy to solve the problem being explored.

Addition & Subtraction Strategies Growth Points

The ENRP Addition & Subtraction Growth Points (Figure 5) provide teachers with a useful framework for evaluating the strategies a child uses to solve numerical problems. This information can be used as a starting point in planning future instructional activities that meet the learning needs of each child.

These growth points describe the learning trajectory and transitions in thinking as children develop arithmetic strategies. Assessment tasks that suggest the positions children have reached along this learning trajectory and identify the growth points they have developed, enable teachers to provide instructional activities that will provoke and support children move towards the next growth point in their learning.

1. Not apparent
   - Not yet able to combine and count two collections of objects.
2. Count all
   - Counts all to find the total of two collections.
3. Count on
   - Counts on from one number to find the total of two collections.
4. Count back/count down to/count up from
   - Given a subtraction situation, chooses appropriately from strategies including count back, count down to and count up from.
5. Basic strategies (doubles, commutativity, adding 10, tens facts, other known facts)
   - Given an addition or subtraction problem, strategies such as doubles, commutativity, adding 10, tens facts, and other known facts are evident.
6. Derived strategies (near doubles, adding 9, build to next ten, fact families, intuitive strategies)
   - Given an addition or subtraction problem, strategies such as near doubles, adding 9, build to next ten, fact families and intuitive strategies are evident.
7. Extending and applying addition and subtraction using basic, derived and intuitive strategies
   - Given a range of tasks (including multi-digit numbers), can solve them mentally, using the appropriate strategies and a clear understanding of key concepts.

Figure 5. ENRP Addition & subtraction growth points
In order to identify the strategies a child uses to solve a number problem, and therefore locate the positions children have reached along this learning, consider the following scenario. A child playing Snakes & Ladders wants to move a counter the number of places represented by the dice in Figure 6.

![Dice](image)

**Figure 6:** Dice representing the number of places to move during a game of Snakes and Ladders

The number of places to move on the *Snakes and Ladders* board may be determined in a variety of ways. One way of evaluating the strategies used to solve this problem, is to consider the strategies in relation to the Addition and Subtraction Growth Points. Each growth point, described below, represents an increasing power in working with number and an important transition in children’s thinking.

**Growth Point 1: Count-all**

Children who have reached this growth point may be limited to counting those items they can see, feel, hear, etc. Therefore, in order to solve the dice problem they may point in turn to each dot on the dice to count the total. Having looked at the dice, if one die was then covered so that the child could no longer see it, he/she would be unable to work out how many places to move. Some children may be no longer dependent on direct sensory input to count items in a group, but typically need to re-present a sensory experience when counting. For example, imagine a child sees five dots on a die that is then covered, and three dots on another die that is then also covered. In order to find the total on the two dice the child may need to represent the situation with counters, use fingers to represent the dots, visualise the blocks and count the visual image, or use movements to represent the blocks and count each movement. A strategy common to this growth point is that children will count all the dots on the two dice and will not count-on; the counting will always begin from one.

**Growth Point 2: Count-on**

Typically, children who have reached Growth Point 2 are now able to count-on to solve addition and missing addend problems involving 'screened' collections. This count is an abstract one since the count takes place without reference to actual objects. Therefore, if a child sees the dice described earlier and the die displaying five dots is covered, in order to find the total on the two dice, the child is able to count-on from five.

**Growth Point 3: Count-back/count-down-to/count-up-from**

Children at this stage are able to count-on, count-back, count-down-to, and count-up-from, choosing the most appropriate strategy to solve problems. Children can conceive of a number sequence being broken into two parts. Hence, if a child knows that the total of two dice is 7 and that one die displays 4 dots, the child can think of the sequence being broken at 4, and count backwards from 7 to 4 to determine the unknown number of dots on the remaining die. Alternatively, the child might count forward from 4 to 7, which may be easier. Additionally, children who have reached this growth point are able to use count-down-to strategies to solve subtraction tasks, such as: 8 - 5 = _. Such tasks are achieved through counting along a number sequence in the required direction, from a convenient starting point (for example, 8, 7, 6, 5 or 7, 6, 5).
Growth Point 4: Basic strategies (doubles, commutativity, adding 10, tens facts, other known facts)

Children who have reached this growth point are no longer dependent on counting by ones to solve arithmetic problems, but have more powerful strategies in their repertoires. Children who have reached this growth point are simultaneously aware of two number sequences and understand that addition and subtraction are inverse operations. In order to solve the 'dice' problem, children use a range of strategies, such as commutativity, subitising (recognising visual patterns), using addition to work out subtraction, and using known facts such as doubles and number facts for ten.

Growth Point 5: Derived strategies (near doubles, adding 9, build to next ten, fact families, intuitive strategies)

Given more complex addition or subtraction problems, children who have reached this growth point use strategies such as near doubles, adding 9, building to the next ten, and fact families. For example, if a child has reached 36 on the Snakes and Ladders board and then rolls a total of 9 on the dice, to work out what number the marker would land on, he or she might build to the next ten (40) and then add 5, or perhaps add 10 and subtract 1 to reach 45 (adding 9 strategy). If the child rolls a 4 and a 5, a near doubles strategy may be used to find the total.

These growth points describe a possible pathway for children’s learning. Identifying the growth point a child has reached guides the teacher’s development of instructional activities to assist the child reach the next growth point in their learning, and thus informs the teacher’s response.

MULTIPLICATION & DIVISION STRATEGIES

The ENRP Multiplication and Division Growth Points (Figure 7) describe the learning trajectory as children develop multiplication and division strategies. Assessment tasks that suggest the positions children have reached along this learning trajectory and identify the growth points they have developed, enable teachers to provide instructional activities that will provoke and support children move towards the next growth point in their learning.

0. Not apparent
   Not yet able to create and count the total of several small groups.
1. Counting group items as ones
   To find the total in a multiple group situation, refers to individual items only.
2. Modelling multiplication and division (all objects perceived)
   Models all objects to solve multiplicative and sharing situations.
3. Abstracting multiplication and division
   Solves multiplication and division problems where objects are not all modelled or perceived.
4. Basic, derived and intuitive strategies for multiplication
   Can solve a range of multiplication problems using strategies such as commutativity, skip counting and building up from known facts.
5. Basic, derived and intuitive strategies for division
   Can solve a range of division problems using strategies such as fact families and building up from known facts.
6. Extending and applying multiplication and division
   Can solve a range of multiplication and division problems (including multi-digit numbers) in practical contexts.

Figure 7. ENRP Multiplication & division growth points

In order to identify the strategies a child uses to solve a multiplication or division problem, and therefore locate the positions children have reached along this learning trajectory, consider the following description of the growth points.
Growth Point 1. Counting group items as ones (Counts one by one without reference to the group structure)

Children who have reached this growth point are able to work out successfully the answer to a problem such as, how many wheels are there on 4 bicycles? However, a child will count the wheels by ones without any reference to the group structure. Similarly, when sharing eight pencils between four children, the child will share by ones, without grouping.

Growth Point 2. Modelling multiplication and division when all objects perceived (uses grouping strategies with visual or physical modelling, e.g., skip counting, repeated addition/subtraction, number facts).

At this growth point, children use the group structure to assist their problem solving. However, they require visual or physical modelling to support their strategies. In solving the earlier bicycle problem, children will either refer to the model and skip count from two to eight, or will add a series of twos or fours, or may use the number fact 4 twos are eight. In the division problem described earlier, children will use strategies such as sharing in groups of two.

Growth Point 3. Abstracting multiplication and division (uses grouping strategies without visual or physical modelling e.g., skip counting, repeated addition/subtraction, number facts.)

Children who have reached the abstract strategies growth point are able to solve multiplication and division problems without reference to models of the problems. They will use skip counting, repeated addition or subtraction, or number facts to solve problems such as, how many days altogether in three weeks? or how many people are in the room if I can see 16 shoes?

Growth Point 4. Basic, derived and intuitive strategies for multiplication. (Can recall known multiplication number combinations; uses multiplication and division as inverse relationships and commutativity.)

At this growth point, children are able to solve problems through recalling number facts. Additionally, they may recognise that they can use one known number fact, such as 7x2=14, to solve an unknown fact with numbers in the reverse order, such as 2x7=14. Children who have reached this growth point are able also to use a known fact, such as 3x5=15 to work out that 3x50=150. Children are also able to extend known facts, such as 3x7=21 to work out unknown facts, such as 6x7=42, by doubling the former answer. Further, a child may solve 9x8, by using 10x8=80 and subtracting eight to determine the answer.

Growth Point 5. Basic, derived and intuitive strategies for division. Extended strategies (uses known facts to multiply and divide by multiples of 10 and 100, extends known facts to work out unknown facts).

At this growth point, children are able to solve division problems through recalling number facts. They may use known multiplication facts such as 7x8=56 to solve division problems such as 56÷8=n, or they may use 8+2=4 to work out 800÷200=4. Similarly, a child may solve 72÷8, by recognising that the answer is one group of eight less than the answer to 80÷8.

Growth Point 6. Extending and applying multiplication and division

Can solve a range of multiplication and division problems (including multi-digit numbers) in practical contexts.
BUILDING VISUAL & PHYSICAL REPRESENTATIONS OF NUMBERS

One strategy to assist children develop number sense is provoking the development of powerful visual images of numbers. In solving the dice problem presented in our scenario above, most people look at the visual patterns shown on the dice and instantly recognise the patterns as ‘four’ and ‘three’ without having to count at all. This ability to subitise is an important skill to develop. It is necessary for teachers to spend time helping children become efficient at instantly recognising the numerical value of a range of visual patterns. Flash cards are useful in this regard, as are card games that require children to match visual patterns with numerals or number words. Games using the visual patterns on dice are also invaluable. During such activities, children must be encouraged to state the numerical value of the visual pattern without reverting to less efficient strategies, such as counting by ones.

Using ten frames to build visual & physical representations of numbers

Another effective tool to help children develop both visual images and physical representations of numbers is the ten frame. The physical representation of numbers on ten frames become powerful visual images to assist children in solving problems. Figure 8 shows how numbers can be represented on a ten frame. Children may physically represent the number using a ten frame mat and objects such as counters or beans, or they may represent numbers on ten frame templates through drawing. Ten frames can be made into place mats to rest on tables or the floor. Large ten frames big enough for children to sit or stand in can be made from fabric or shade cloth, or marked out on the floor with masking tape.

Use of the ten frame assists children develop a visual image of a number that can become automatically recognised. The automatic recognition of numbers represented by visual patterns is a useful skill to develop. This is what most people do when using dice or dominoes. It is much easier to look at a collection and know that there are six, than to have to count the collection. Using ten frames to represent numbers also assists children to think about numbers in reference to ten. This supports children both learn and derive number facts.

![Ten Frames](image)

*Figure 8: Representations of one, three, four, six, seven, and nine on 'tens frames.'*

When exploring the representation of particular numbers on a ten frame, children may be asked to represent numbers in as many ways as possible. However, they can also be challenged to identify which representation of the number is the easiest to automatically recognise. Figure 9 shows different representations of the number four. Some representations are instantly recognisable as four and others require strategies such as counting or grouping in order to work out the total number in the collection.
Building numerical part/whole relations

The ten frame is useful for developing images of numbers which support the development of numerical part/whole relations. Children can use a ten frame to explore the many ways a number may be partitioned and the parts re-combined to make the whole. The use of colour can facilitate the representation of the parts.

Building images of tens and ones

The ten frame can be used to assist children develop images of two digit numbers as groups of tens and ones. Figure 10 shows a representation of sixteen as a ten and a group of six. To assist the development of place value ideas, children can be encouraged to describe teen numbers such as sixteen as a ten and six ones.

Ten frames may also be used effectively to help children develop visual images for counting on and counting back, for modeling doubles and near doubles facts, for adding and subtracting, and for modeling build to ten strategies.

Think board

The Think Board is a useful way to cater for the different learning and thinking styles of children. It also assists children make connections between the language that is used to describe a numerical situation they are exploring, a physical representation of the situation, a visual representation of the situation and the way the situation may be represented symbolically. The Think Board is partitioned into four sections; one for each of the language, physical, visual and symbolic representations of the situation being explored. Versions of the Think Board may be large enough for children to gather around on the floor, or small enough to place on children’s work tables.
Figure 11 provides an example of how the Think Board may be used to represent the following situation: Nga built a tower that was 3 blocks higher than Clare’s tower. How many blocks high might the two towers be? In the real situation segment of the Think Board, a written description of the situation is recorded or discussed. Placed in the materials segment of the Think Board are two block towers that differ in height by three blocks. This solution shows that one tower is nine blocks high and the other tower is twelve blocks high. The picture segment shows how a drawing may be used to visually represent the two towers and helps children develop a mental image of the situation. Finally, the symbolic segment shows how written symbols may be used to describe the situation. Therefore, the Think Board gives children the opportunity to use and make links between four different ways of representing number situations.

Figure 12 provides an example of how the Think Board may be used to represent a number situation: Lisa collected forty-seven beans. In the real situation segment of the Think Board, a written description of the situation is recorded or discussed. Placed in the materials segment of the Think Board are objects representing the forty-seven beans. In this case the actual beans are used. The picture segment shows how a drawing may be used to visually represent the collection of objects and helps children develop a mental image of the situation. Finally, the symbolic segment shows how written symbols may be used to describe the situation. Therefore, the Think Board gives children the opportunity to use and make links between four different ways of representing number situations.

CONCLUSION

Counting, reading, writing and interpreting numerals, arithmetic strategies and visualisation are all important aspects of children’s early number learning. In order to effectively provoke children’s learning in each of these areas, teachers need a clear understanding of how children learn number concepts and skills, knowledge of the growth points reached by each child in the class in relation to a conjectured learning trajectory, and an awareness of the difficulties children may face when learning number ideas. Effective teaching takes account of this information when planning and implementing instructional activities to provoke each child’s number learning. Inducing children to develop visual images of number ideas and situations using resources such as ten frames and the Think Board, is another important way to assist children develop number sense, confidence and strategies for solving the numerical problems they encounter each day.

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